MAT 616 Notes about proofs 2/15/2022

1 What are proofs in the context of this class

Proofs have many objectives. They are commonly thought as a way to corroborate to oneself and others that a statement is accurate. We will mostly look at Mathematical proofs and Story Telling as a way to convice oneself others of a statement.

We will classify proofs into three large groups: Mathematical proofs , Retorical proofs and the rest (which includes any crazy thing human beings may do to convince themselves and make another person believe something is true).

In Rhetoric, proof is the part of a speech or written composition that sets out the arguments in support of a thesis. Also known as confirmation, confirmatio, pistis, and probatio. In classical rhetoric, the three modes of rhetorical (or artistic) proof are ethos, pathos, and logos. We will not talk much about this in this class, but we should be aware of the field of rethoric.

Story Telling in Data science is a way to encase information in a format that is digestible to an audience, and can be thought as a way to justify, explain or prove that something is accurate. Even is a story is science fiction, it is convincing in the way that it accurately portrais human values and cognitive truth. If a story does not include cognitive valid and existing facts, it probably cannot understood by the audience. For instance, noise is experiencial signals that have no cognitive reality embedded in it. Story Telling is not a proofs, and it only seeks to approach a proof better and better. In Data Science, Story Telling can always be improved (interestingly enough) by even by changing the data used in the first place.

Writing proofs of statements has the following benefits and properties:

1. A proof gives a better understanding of why and in which sense the statement is true.
2. Mathematical proofs are a particular type of proof, which uses formal language and has an extremely well developed infrastructure.
3. A Mathematical proof can only be created if the statement to be proved is correctly stated (more about "correctly stated" this later).
4. A proof clarifies and elaborates on what the statement is saying.
5. A proofs makes it easier to remember and use the statement.
6. Mathematical proofs are a sequence of statements, starting at the original statement's assumptions and ending at the original statement's claim.

2 Example of a mathematical proof

Theorem MT-even: In the MuMuMuu theory (MT), the number of u's is always even.

Proof: All the words in MT are derived from MuMuMuu by using rules 2 and 3. The original word has 4 u's, which is even. Rule 2 adds 2 u's and Rule 3 does not add any u's. Hence the theorem follows. QED

Comments: Note that Mathematical proofs:

- assume that the reader understands the statement and the assumptions, even if not fully state.
- assume that the reader knows the field.
- do not always review content material, but only refers to it. Proofs assume people can look up anything needed, as long as it is mentioned.

3 Equations

Definition: An algebraic equation is a statement containing numbers, constants and variables on both sides on an equal sign.

Equations Rule 1: If an equation is true, you can add the same thing(s) to both sides or multiply both sides by the same thing(s) to create a new true equation.

Equations Rule 2: If an equation is true, you can swap the left and right side to create a new true equation.

HW: Consider the following (incorrect?) reasoning

$a = b$	1. a and b are assumed to be the same real number
$ab = bb$	2. Rule 1
$ab - a^2 = b^2 - a^2$	3. Rule 1
$a(b-a) = (b-a)(b+a)$	4. Expand (b-a)(b+a) to get b^2 - a^2
$a(b-a)\frac{1}{b-a} = \frac{1}{b-a}(b-a)(b+a)$	5. Rule1
$a = b + a$	6. Note that (b-a)/(b-a) equals 1, i.e. cancel the (b-a)
$a = 2a$	7. Since b = a, replace b by a
$\frac{1}{a}a = \frac{1}{a}2a$	8. Rule 1
$1 = 2$	9. WHAT??? This cannot be true!!!

Question 1: How many new equations were created from the assumed equation (a = b)?

Question 2: How many of the new equations are proven to be True by using E-Rules and basic math manipulations? Explain briefly and accurately your answer.

Question 3: Suppose you start with a false equation and use E-Rules 1 and 2 to get a new equation. Explain what the possible scenarios are.